Exam

Name_____

1) Let $y = f(x)$ and $\frac{d^2y}{dx^2} = y'' = x(x+2)^2$. Then the graph of f is 1) _____

concave up for

A) $x < 0$

B) $-2 < x < 0$

C) $x > 0$

D) $x > 4$

E) $-\infty < x < \infty$

2) Let $y = f(x)$ and $\frac{d^2y}{dx^2} = y'' = x(x+1)(x-2)^2$. Then the graph of f has 2) _____

inflection points when x =

A) 2

B) - 1 and 0

C) - 1, 2, 3

D) 1 and 2

E) 1

3) Let $y = x^4 - 2x^2 + 5$. Determine the points (if any) at which the graph of the function has a **vertical tangent line.** 3) _____

A) No vertical tangent line

B) $x = -3, 0, 2$

C) $x = \frac{1}{2}, -\frac{2}{3}$

D) $x = -1, 0, 1$

E) $x = -\sqrt{2}, \sqrt{2},$

4) Let $y = f(x) = \frac{x^4 + 1}{x^2}$. Then the graph of f is **decreasing** on 4) _____

A) $(-\infty, \infty)$
B) $(-1, 0)$
C) $(-\infty, -1) \cup (0, 1)$
D) $(1, \infty)$
E) $(2, 3) \cup (7, \infty)$

5) Let $y = x^2 - 2x$ on $[-1, 2]$. The **maximum value** occurs at 5) _____

A) $x = 0$
B) $x = 2$
C) $x = 1$
D) $x = -1$
E) $x = -\frac{1}{2}$

6) Which one of the following is **false** ? 6) _____

A) Continuity at a point implies differentiability at that point
B) $\frac{d}{dx} \ln(x^3 + 1) = \frac{3x^2}{x^3 + 1}$
C) $\frac{d}{dx} (\sin(x^2)) = 2x \cos(x^2)$
D) $\frac{d}{dx} (x^2) = 2x$
E) $\frac{d}{dx} (7) = 0$

7) Let $y = \left(\frac{x+1}{x+3}\right)^4$. Then $\frac{dy}{dx}$ is equal to (in simplified form) 7) _____

A) $8\,\frac{(x+1)^3}{(x+3)^5}$

B) $\frac{x+1}{x+3}$

C) $\frac{(x-3)^3}{(x-1)^5}$

D) $4\,\left(\frac{x+1}{x+3}\right)^3$

E) $8\,x$

8) Let $y = \sin(xy)$. Calculate $\frac{dy}{dx}$? 8) _____

A) $\frac{dy}{dx} = \cos(xy)$

B) $\frac{dy}{dx} = \frac{y}{1 - x\cos(xy)}$

C) $\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$

D) $\frac{dy}{dx} = \frac{1}{1-x}$

E) $\frac{dy}{dx} = \frac{y\cos(xy)}{1-x}$

9) Let $y = f(x) = \sin(x) - 1$, $0 < x < 2\pi$. Then the graph of f is **increasing** on 9) _____

A) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

B) $(0, \pi)$

C) $(0, 2\pi)$

D) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

E) $\left(\pi, \frac{3\pi}{2}\right)$

10) Let $y = x\cos(x)$. Calculate $\frac{dy}{dx}\left(\frac{\pi}{4}\right)$, that is, the value of the first derivative at $x = \frac{\pi}{4}$. ? **Hint** : Calculate $\frac{dy}{dx}$ and replace x with $\frac{\pi}{4}$. 10) _____

A) $2\left(1 - \frac{\pi}{2}\right)$

B) $2\left(1 + \frac{\pi}{2}\right)$

C) 1

D) $\frac{\sqrt{2}}{2}\left(1 - \frac{\pi}{4}\right)$

E) $\frac{\sqrt{2}}{2}$

11) Let $y = x^x$. To find $\frac{dy}{dx}$, which method should be used ? 11) _____

A) Product rule

B) Logarithmic differentiation

C) Implicit differentiation

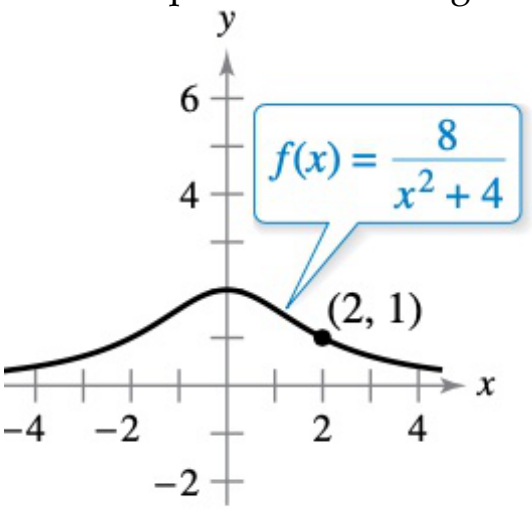
D) Power rule

E) Quotient rule

12) All edges of a cube are expanding at a rate of 6 centimeters per second. How fast is the volume changing when each edge is 2 centimeters. **Hint** : Volume of a cube is $V = x^3$, where x is edge lenght of cube. 12) _____

A) 18 B) 72 C) 12 D) 36 E) 144

13) Find an equation of the tangent line to the graph at the given point ? 13) _____



A) $y = \frac{1}{2}x$

B) $y = -\frac{1}{2}x + 7$

C) $y = -x + 2$

D) $y = -\frac{1}{2}x + 2$

E) $y = 2x + 2$

14) Let $xy = 4$ and assume that x and y are both differentiable functions of t. Calculate $\frac{dx}{dt}$ when $x = 1$ and $\frac{dy}{dt} = -6$. 14) _____

A) $-\frac{2}{3}$ B) $\frac{3}{2}$ C) -6 D) 3 E) 2

15) Find k such that the line $y = -6x + 1$ is tangent to the graph of the function $y = k - x^2$? 15) _____

A) -6 B) 0 C) -4 D) 25 E) -8

16) Let $y = \frac{6x^2}{x - 2}$ on $[-2, 1]$. The **maximum value** occurs at 16) _____

A) $x = \frac{1}{2}$

B) $x = -2$

C) $x = 0$

D) $x = 1$

E) $x = -\frac{1}{2}$

17) Let $y = (2x - 7)^3$. Calculate $\frac{dy}{dx}$? 17) _____

A) $\frac{dy}{dx} = e^{2x - 7}$

B) $\frac{dy}{dx} = 6\ (2x - 7)^2$

C) $\frac{dy}{dx} = \frac{1}{2x - 7}$

D) $\frac{dy}{dx} = 2x - 7$

E) $\frac{dy}{dx} = 3.(2x - 7)^{-3}$

18) Find the function $y = y(x)$ such that $y' = \frac{dy}{dx} = -4$ and $y(0) = 2$. 18) _____

A) $y = -4x + 2$

B) $y = -4x + \frac{1}{2}$

C) $y = 2x$

D) $y = -2x$

E) $y = -2x + 4$